WSM 2019 talk on

Multi-Depot Two Echelon Capacitated Vehicle Routing Problem with Heterogeneous Fleet

Surendra Reddy Kancharla

Senior Project Officer
Intelligent Transportation Systems Lab
Indian Institute of Technology Madras

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Introduction

- Two level distribution system - large trucks deliver goods to satellites (UCCs), and then smaller trucks deliver it to customers.
- Freight transport, which contributes to **10-15%** of road traffic, accounts for **40%** of the vehicular air pollution in cities.
- Fuel and driver costs constitute more than **60%** of cost to operators.
- Minimizing fuel is win-win for the operator and environment.

Fuel Consumption Estimation

CMEM model

Engine power required

$$P = \frac{(Ma + Mg\sin\theta + MgC_r\cos\theta + 0.5C_d\rho Av^2)v}{1000\epsilon} + P_{acc}, \quad (1)$$

Fuel rate

$$FR = \frac{\varphi(kNV + P/\eta)}{U}, \quad (2)$$

$$FR = \Gamma + \Xi M, \quad (3)$$

where,

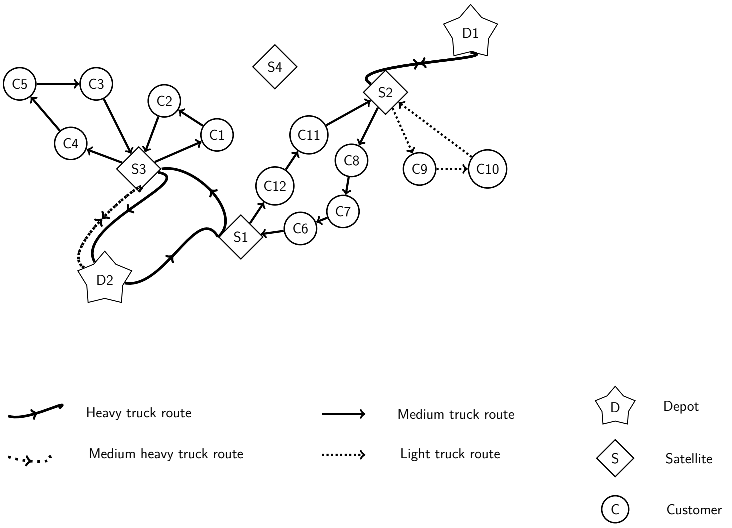
$\Gamma = \dfrac{\varphi\left(kNV + \dfrac{0.5C_d\rho Av^3}{1000\,\eta\,\epsilon}\right)}{U}$ is the weight independent part, and

$\Xi M = \dfrac{\varphi(a + g\sin\theta + gC_r\cos\theta)v}{1000\,\epsilon\,\eta\,U}M$ is the weight dependent part.

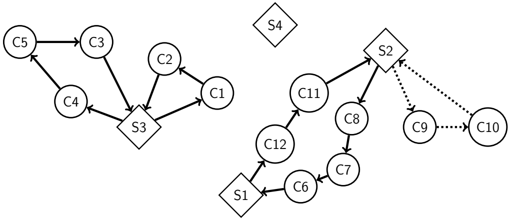
Data Requirement

- Speed is governed by the traffic state around- can't use fixed speed
- Driving cycles - speed time profile of a representative vehicle for a given area
- CMEM model with Driving cycles can give better fuel consumption estimates than fixed speed
- Huge GPS data set allows development of driving cycles at link level

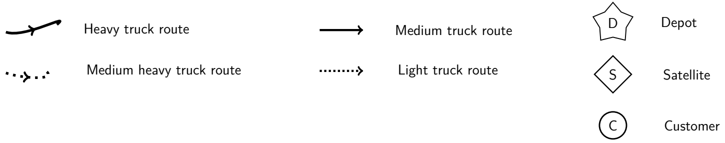
MD2E-CVRP


D1
S4
C5
C3
C2
C1
S2
C11
C4
S3
C8
C9
C10
C12
C7
S1
C6
D2
Heavy truck route
Medium truck route
Medium heavy truck route
Light truck route
D
Depot
S
Satellite
C
Customer

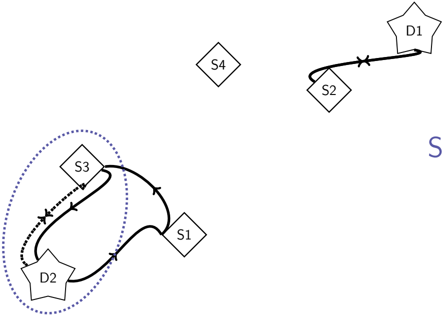
MD2E-CVRP



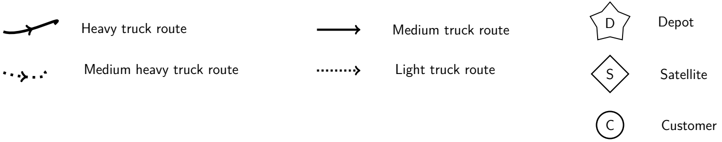
Location routing problem



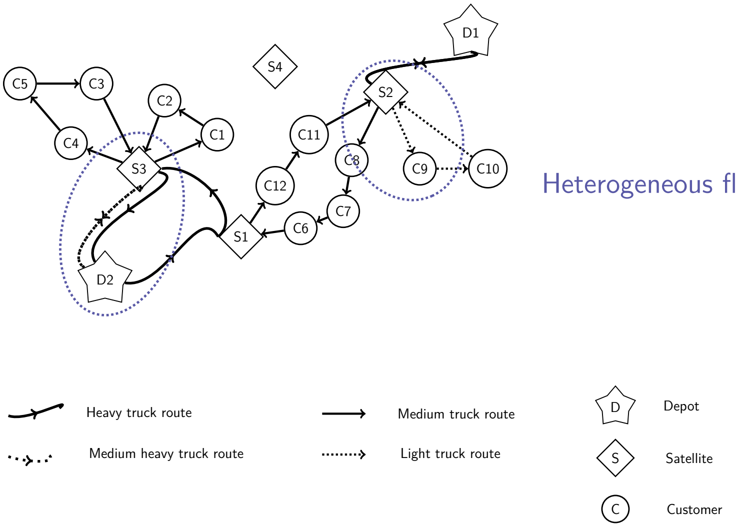
MD2E-CVRP



Split delivery routing problem

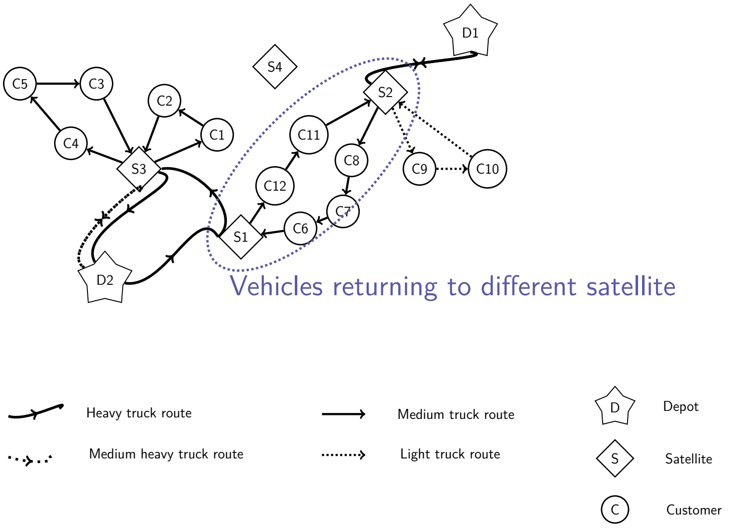


MD2E-CVRP



Heterogeneous fleet

MD2E-CVRP


D1
S4
C5
C3
C2
C1
C4
S3
S2
C11
C8
C9
C10
C12
C7
C6
S1
D2
Vehicles returning to different satellite
Heavy truck route
Medium truck route
Medium heavy truck route
Light truck route
D
Depot
S
Satellite
C
Customer

Mathematical Formulation

Minimize fuel required for both echelons

$$\min \sum_{t\in \mathbf{T}} \sum_{h\in \mathbf{H}} \sum_{(i,j)\in \mathbf{E}_1} (\Gamma_{ij}^{h} + \Xi_{ij}^{h}\psi_h) y_{ij}^{th} + \Xi_{ij}^{h} Q_{ij}^{th} \quad + \sum_{s\in \mathbf{S}} \sum_{w\in \mathbf{W}} \sum_{(k,l)\in \mathbf{E}_2} (\Gamma_{kl}^{w} + \Xi_{kl}^{w}\psi_w) x_{kl}^{sm} + \Xi_{kl}^{w} Q_{kl}^{sm} \tag{4}$$

Subject to:

Limit on vehicles

$$\sum_{j\in \mathbf{S}} y_{tj}^{th} \leq \gamma_t^h, \quad \forall\ t \in \mathbf{T}, h \in \mathbf{H} \tag{5}$$

$$\sum_{l\in \mathbf{C}} x_{sl}^{sm} \leq \gamma_s^w, \quad \forall\ s \in \mathbf{S}, m \in \mathbf{W} \tag{6}$$

Inflow = Outflow @ depot and satellite

$$\sum_{j\in \mathbf{S}} y_{pj}^{ph} = \sum_{t\in \mathbf{T}} \sum_{j\in \mathbf{S}} y_{jp}^{th}, \quad \forall\ p \in \mathbf{T}, h \in \mathbf{H} \tag{7}$$

$$\sum_{l\in \mathbf{C}} x_{pl}^{pm} = \sum_{s\in \mathbf{S}} \sum_{l\in \mathbf{C}} x_{lp}^{sm}, \quad \forall\ p \in \mathbf{S}, m \in \mathbf{W} \tag{8}$$

Inflow = Outflow
@ satellite & customer

$$\sum_{\substack{i\in\mathbf{S}\cup\mathbf{T}\\ i\neq s}} y_{is}^{th} = \sum_{\substack{i\in\mathbf{S}\cup\mathbf{T}\\ i\neq s}} y_{si}^{th}, \quad \forall\, s\in\mathbf{S}, t\in\mathbf{T}, h\in\mathbf{H} \tag{9}$$

$$\sum_{\substack{k\in\mathbf{S}\cup\mathbf{C}\\ k\neq c}} x_{kc}^{sm} = \sum_{\substack{k\in\mathbf{S}\cup\mathbf{C}\\ k\neq c}} x_{ck}^{sm}, \quad \forall\, c\in\mathbf{C}, s\in\mathbf{S}, m\in\mathbf{W} \tag{10}$$

Demand satisfaction
for both echelons

$$\sum_{t\in\mathbf{T}}\sum_{\substack{i\in\mathbf{S}\cup\mathbf{T}\\ i\neq s}} Q_{is}^{th} - \sum_{t\in\mathbf{T}}\sum_{\substack{i\in\mathbf{S}\cup\mathbf{T}\\ i\neq s}} Q_{si}^{th} = \sum_{t\in\mathbf{T}} \omega_{ts}^{h}, \quad \forall\, s\in\mathbf{S}, h\in\mathbf{H} \tag{11}$$

$$\sum_{s\in\mathbf{S}} Q_{ts}^{th} = \sum_{s\in\mathbf{S}} \omega_{ts}^{h}, \quad \forall\, t\in\mathbf{T}, h\in\mathbf{H} \tag{12}$$

$$y_{ij}^{th} \leq \Omega Q_{ij}^{th}, \quad \forall\, t\in\mathbf{T}, h\in\mathbf{H}, (i,j)\in\mathbf{E}_1 \tag{13}$$

$$\sum_{h\in\mathbf{H}}\sum_{t\in\mathbf{T}}\sum_{j\in\mathbf{S}} Q_{jp}^{th} = 0, \quad \forall\, p\in\mathbf{T} \tag{14}$$

$$\sum_{s\in\mathbf{S}}\sum_{w\in\mathbf{W}}\sum_{\substack{k\in\mathbf{S}\cup\mathbf{C}\\ k\neq c}} Q_{kc}^{sm} - \sum_{s\in\mathbf{S}}\sum_{w\in\mathbf{W}}\sum_{\substack{k\in\mathbf{S}\cup\mathbf{C}\\ k\neq c}} Q_{ck}^{sm} = d_c, \quad \forall\, c\in\mathbf{C} \tag{15}$$

$$\sum_{w\in\mathbf{W}}\sum_{s\in\mathbf{S}}\sum_{k\in\mathbf{C}} Q_{kp}^{sm} = 0, \quad \forall\, p\in\mathbf{S} \tag{16}$$

Can not carry more than capacity of vehicle

$$Q_{ij}^{th} \leq \zeta_h y_{ij}^{th}, \quad \forall\ t \in \mathbf{T}, h \in \mathbf{H}, (i,j) \in \mathbf{E}_1 \tag{17}$$

$$Q_{sl}^{sm} \leq \zeta_w x_{sl}^{sm}, \quad \forall\ l \in \mathbf{C}, s \in \mathbf{S}, w \in \mathbf{W} \tag{18}$$

Visit each customer once

$$\sum_{m \in \mathbf{W}} \sum_{s \in \mathbf{S}} \sum_{k \in \mathbf{S} \cup \mathbf{C}} x_{kc}^{sm} = 1, \quad \forall\ c \in \mathbf{C} \tag{19}$$

Link between two echelons

$$\sum_{t \in \mathbf{T}} \sum_{h \in \mathbf{H}} \omega_{ts}^{h} = \sum_{w \in \mathbf{W}} \sum_{k \in \mathbf{S} \cup \mathbf{C}} \sum_{c \in \mathbf{C}} x_{kc}^{sm} d_c, \quad \forall\ s \in \mathbf{S} \tag{20}$$

Domain of decision variables

$$y_{ij}^{th} \in \mathbb{Z}^+, \quad \forall\ t \in \mathbf{T}, h \in \mathbf{H}, (i,j) \in \mathbf{E}_1 \tag{21}$$

$$x_{kl}^{sm} \in \{0,1\}, \quad \forall\ s \in \mathbf{S}, w \in \mathbf{W}, (k,l) \in \mathbf{E}_2 \tag{22}$$

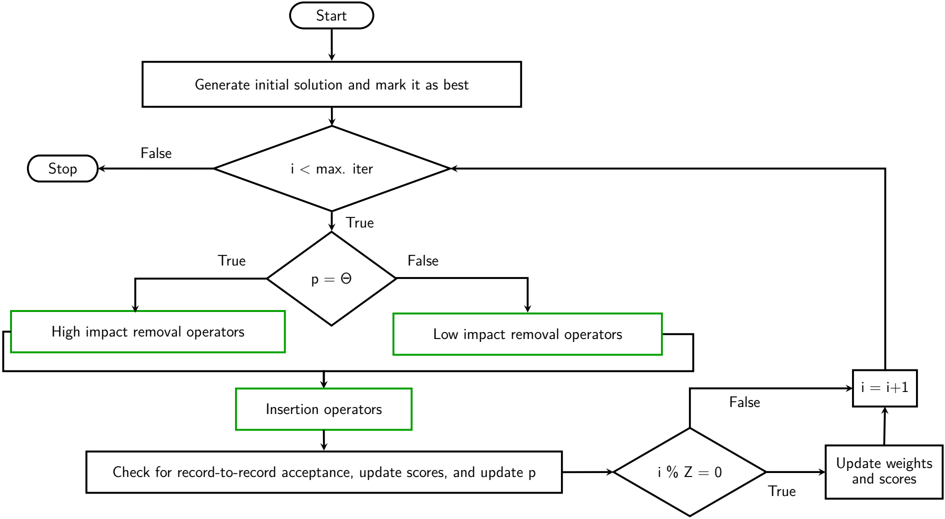
$$\omega_{tj}^{h} \in \mathbb{R}^+, \quad \forall\ t \in \mathbf{T}, h \in \mathbf{H}, j \in \mathbf{S} \tag{23}$$

$$Q_{ij}^{th} \in \mathbb{R}^+, \quad \forall\ t \in \mathbf{T}, h \in \mathbf{H}, (i,j) \in \mathbf{E}_1 \tag{24}$$

$$Q_{kl}^{sm} \in \mathbb{R}^+, \quad \forall\ s \in \mathbf{S}, w \in \mathbf{W}, (k,l) \in \mathbf{E}_2 \tag{25}$$

ALNS Algorithm

- Ropke and Pisinger (2006) introduced Adaptive Large Neighborhood Search (ALNS), which extends the large neighborhood search.
- It works on the principle of ruin-and-recreate, and requires an initial solution to start.
- The removal and insertion operators are selected based on their past success using the roulette wheel selection method.
- Solution acceptance criteria:
  - Record-to-record acceptance
  - Simulated annealing acceptance


Start
Generate initial solution and mark it as best
i < max. iter
False
Stop
True
p = Θ
True
False
High impact removal operators
Low impact removal operators
Insertion operators
Check for record-to-record acceptance, update scores, and update p
i % Z = 0
False
True
i = i+1
Update weights and scores

Experiments

- MILP is coded in GAMS and solved using Gurobi hosted on NEOS server (3GB RAM limit, 8h time limit).
- ALNS is coded in Python and tested on PC with 2.2 GHz Intel processor with 8 GB RAM.
- Instance sets used are as follows:

Instance set	Customers	Satellites	Depots	Distribution of nodes
Set-2	22-51	2-4	1	R, C
Set-3	22-51	2	1	R, C
Set-4	50	2-5	1	RC
Mod set-4	50	3-5	1-3	RC
New set	16-36	3-6	1-3	R, C

R- Random; C- Clustered; RC- Random and clustered

Set-2, set-3 and set-4 are from Perboli et al. 2011

Mod set-4 and new set are from Kancharla and Ramadurai 2019

Results

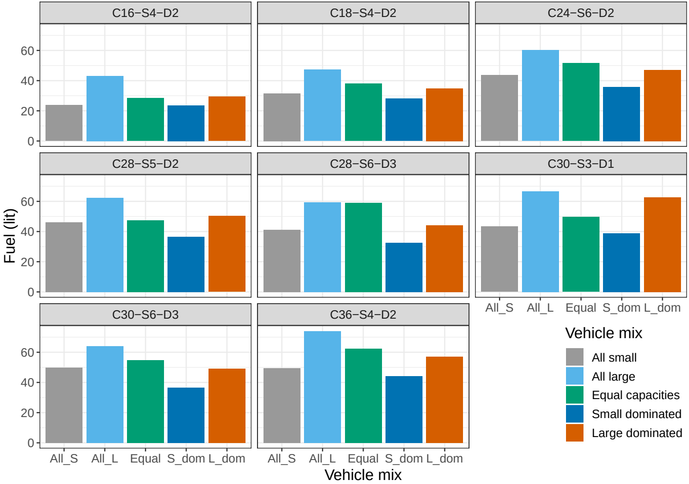
Gurobi results

- Improvements due to relaxation of "returning to same depot/satellite" assumption.

Instance set	Objective	Total Instances	Improvement(%)
Set-2	distance	21	8.06
Set-3	distance	18	7.69
New set	distance	8	11.66
New set	fuel	8	4.51

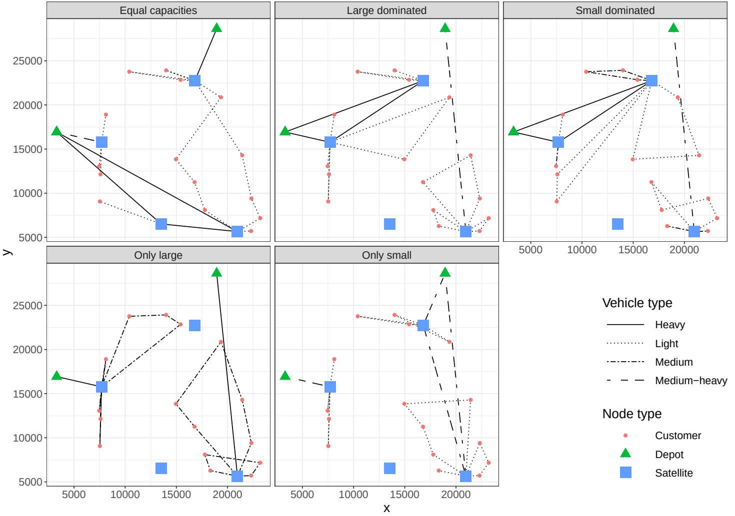
- Fuel minimization led to **13%** savings in fuel despite **15%** increase in distance traveled.

Variation in cost with fleet composition



- Small dominated vehicle mix results in the best fuel cost.

Optimal routes for instance C16-S4-D2



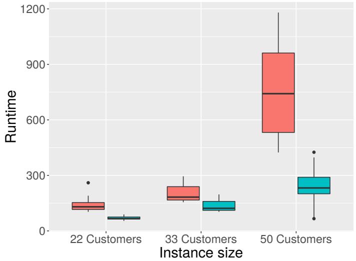
- Small dominated case uses a better mix of available vehicles.

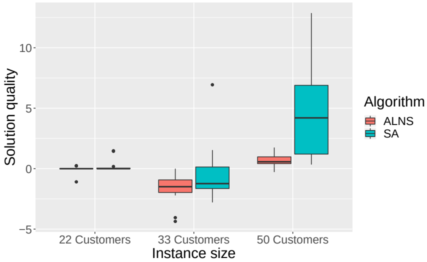
ALNS results

Instance set	Objective	Total Instances	Gurobi	ALNS	Avg. Gap (%)	ALNS Time (s)
Set-2	distance	21	574.23	564.74	1.52	409
Set-3	distance	18	645.73	640.98	0.63	389
Set-4	distance	54	1556.13	1411.38	7.26	771
Mod Set-4	distance	18	1295.5	1009.02	20.43	817
Mod Set-4	fuel	18	25.14	24.46	3.3	893
New set	distance	8	223.63	223.63	0	597
New set	fuel	8	48.88	46.81	3.99	606

- ALNS either improved or matched the BKS in
  - **86** out of **93** 2E-CVRP instances and
  - **29** out of **34** MD2E-CVRP instances.

- ALNS with both vehicles and fuel minimization resulted in **4.31% ↓** in vehicles with **2.71% ↑** in fuel compared to fuel minimization..





- SA is approximately **60%** faster than ALNS with **4%** worse solution quality.

Selected References

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5. Jepsen, M., Spoorendonk, S. & Ropke, S., 2013. A Branch-and-Cut Algorithm for the Symmetric Two-Echelon Capacitated Vehicle Routing Problem. Transportation Science, 47(1), pp.23–37.

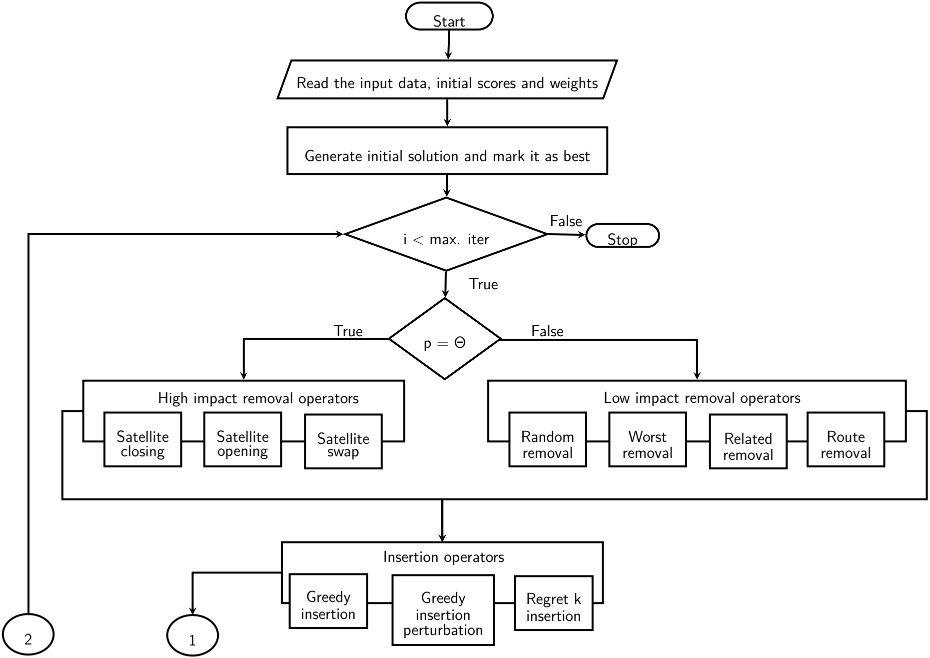
6. Soysal, M., M.Bloemhof-Ruwaard, J. & Bektaș, T., 2015. The time-dependent two-echelon capacitated vehicle routing problem with environmental considerations. International Journal of Production Economics, 164, pp.366–378.

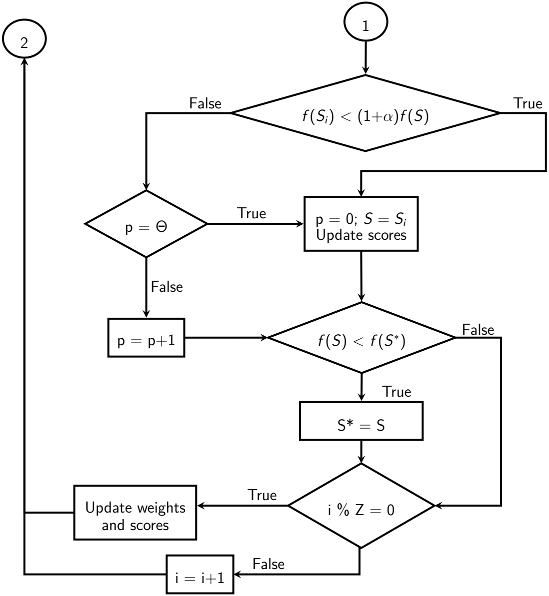
7. Wang, K., Lan, S. & Zhao, Y., 2017. A genetic-algorithm-based approach to the two-echelon capacitated vehicle routing problem with stochastic demands in logistics service. Journal of the Operational Research Society, 68(11), pp.1409–1421.

Thank you ☺

Questions ?

ALNS for MD2E-CVRP




1
2
$f(S_i) < (1+\alpha)f(S)$
False
True
p = Θ
True
p = 0; $S = S_i$
Update scores
False
p = p+1
$f(S) < f(S^*)$
False
True
S* = S
i % Z = 0
True
Update weights
and scores
False
i = i+1